

# **EXPLORATION OF NON –DARCY EMHD BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER A NONLINEAR LEAKY VERTICALLY STRETCHING SHEET WITH TOTAL UPSHOT OF HYDRODYNAMIC AND THERMAL SLIP IN A SOAKED POROUS MEDIUM**

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## **ABSTRACT**

Numerical solutions of the equations of motion and energy for MHD boundary layer viscous incompressible fluid flow and heat transfer with combined effect of hydrodynamic and thermal slip over a permeable nonlinear stretching sheet in a drenched absorbent medium is obtainable in the current work. The electrically conducting fluid occupies a partially beyond measure absorbent space. The non-linear partial

differential equations and boundary conditions are condensed to a system of non-linear ordinary differential equations and boundary conditions by similarity transformations. Runge-Kutta shooting method is implemented to solve the reduced system of equations. Graphs are depicted for dissimilar values of the budding parameters, such as stretching parameter  $n$ , magnetic parameter  $M$ , Electric Parameter  $E_1$ , porosity parameter  $f_w$ , buoyancy parameter  $\lambda$ , Prandtl number  $Pr$ , Eckert number  $Ec$ , hydrodynamic slip parameter  $\gamma$ , porous parameter  $N_1$ , Inertia Coefficient  $N_2$  and thermal slip parameter  $\delta$  and discussed.

*Key words:* Hydrodynamic slip; Thermal slip; Nonlinear stretching; Buoyancy parameter; Electric parameter.

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## INTRODUCTION

In prospect of tremendous applications in engineering automated processes, the problem of viscous fluid flow and heat transfer carrying over stretching surfaces has enthralled the notice of fluid dynamics experts, for decades, has been the region under discussion of enormous curiosity in the accessible literature.

Some of the applications of flow past a stretching surface are, chemical and mechanized processes, hot rolling, paper production, metal revolving, drawing plastic films, glass blowing, annealing and tinning of copper wires, nonstop casting of metals and whirling of fibers, foodstuff dispensation, unbroken casting, portrayal of plastic sheets, and chemical dispensation apparatus, etc.

Electric field is shaped by static charges, where as magnetic field is produced by varying motion of electric charges. The study of magnetic characteristics and behaviour of electrically conducting fluids is known as MHD. MHD has many applications, and to exemplify a few of them involves its applications in medical sciences, the transportation industry and engineering industry. To exhibit this feature one can see that its usage, viz a high intensity electric field is used to diminish or slow down the augmentation of tumour in the brain. MHD absorbs energy and depicts controllable behaviour which makes it useful for cooling purposes especially as a cooling material in electrochemistry and chemical engineering. Magnetic field which acts as external force impacts the flow of electrically conducting fluids and this interaction results in production of Lorentz force and accordingly controls fluid motion, and is essential for industrial production, due to electric field. The fluid flow over a continuously poignant surface finds applications in manufacturing processes, crystal growing, liquid films in reduction course of action, etc., (refer, Fisher [1]). The investigations of Sakiadis [2] and Crane [3] are the most well-known investigations in boundary layer theory. The steadiness scrutiny of MHD flow over a stretching surface was well thought-out for study by H.S. Takhar et al [4]. MHD flow and heat transfer over a stretching surface was considered by H.I. Andersson [5]. MHD flow and heat transfer over a stretching surface with prearranged heat flux was considered by M Kumari et al [6]. Similarity solutions for MHD flow over continuously poignant flat plate with hall effects was considered by Watanabe and Pop [7]. MHD flow over stretching surface with suction/Injection was considered by Pop and Na [8]. The MHD flow over absorbent stretching surface with changeable suction/injection was discussed by N. Chaturvedi [9]. MHD flow and heat transfer past a stretching surface in a

leaky medium was considered by G.A. Rao et al [10]. All the above mentioned researchers [4-10] have discussed concerning flow and heat transfer past a stretching sheet with no-slip condition at the surface. But though, in case of fluid being in the form of, particulate such as emulsions, suspensions, foams, and polymer solutions, the no-slip condition is no longer applicable, specially at the micro and nano scale, hence in such cases, one has to switch over to slip condition.

The problem concerns to heat and mass transfer characteristics of the MHD viscous incompressible fluid flow over a leaky stretching sheet with hydrodynamic and thermal slip is considered for investigation by M.Turkyilmazoglu, [11]. He solved the consequential boundary value problem analytically, and described the effects of various parameters on flow and heat transfer characteristics. Analysis to study MHD stagnation point flow and heat transfer over a stretching/shrinking sheet with collective impacts of velocity slip and heat generation/absorption was considered by Samir Kumar Nandy and Tapas Ray Mahapatra [12]. They have obtained the effects of slip and heat generation/absorption on flow and heat transfer and a comparative study was performed between the previously published research and the present work for a special case and are found to be well in agreement with the present study. The problem of nanofluid flow and thermal transport characteristics with hydrodynamic and thermal slip are solved analytically, resulting in providing unique and double solutions and also rigorous mathematical scaling is proposed, facilitating the nano fluid analysis by M.Turkyilmazoglu, [13]. 2D and axisymmetric flows past a stretching surface with combined effects of magnetic field and porosity with second order slip was investigated by E.H Aly and K.Vajravelu [14]. Further the effects of first order slip, second order

slip, magnetic field, and permeability of porous media on flow and thermal characteristics are investigated. The prominent investigation concerned to the effects of partial slip and third grade fluid parameter on flow and heat transport characteristics, are made by Bikas Sahoo et al [15]. Further they have shown that, the effect of third grade fluid parameter is to increase momentum boundary layer thickness and to decrease the thermal boundary layer thickness. Steady two dimensional boundary layer flow and heat transfer over a vertical permeable stretching /shrinking surface was investigated by Alin V Rosca and Ioan Pop [16]. They have studied the effects of suction and mixed convection parameters, on flow and heat transport. Further they have discussed about stability analysis and have exposed that the upper branch solutions are stable, and therefore not physically possible.

The flow and radiation heat transfer of a nanofluid over a stretching sheet with velocity slip and temperature jump in porous medium is well thought-out for study by Liancun Zheng et al [17]. They compared the numerical results with analytical HAM, and are found to be well in agreement with their results, and an analysis of MHD flow and heat transfer over permeable stretching/shrinking surfaces taking into account a second order slip model is well thought-out in their study. The rationale is to obtain analytical solutions for the flow and heat transport valid under different physical conditions. A special thought is specified for the impact of magnetic field on the second order slip flow conditions. The applied magnetic field play a well-known responsibility in controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching/shrinking sheets. Considering the magnetic field effect, a assortment of corporeal properties for the flow and heat transfer over stretching/shrinking sheet were analytically investigated in Turkyilmazoglu [19–20]. Recently, Fang et al. [21] well thought-out the effects of second

order slip on the flow of a shrinking as well as stretching sheet, that was studied by Vajravelu et al. [22].

In case of slip-flow, if the disjoining of individual molecules occurs at the nanoscales, then one can explain main transport phenomena in nano fluidic systems with a theory depending on continuum and mean-field approaches [23].

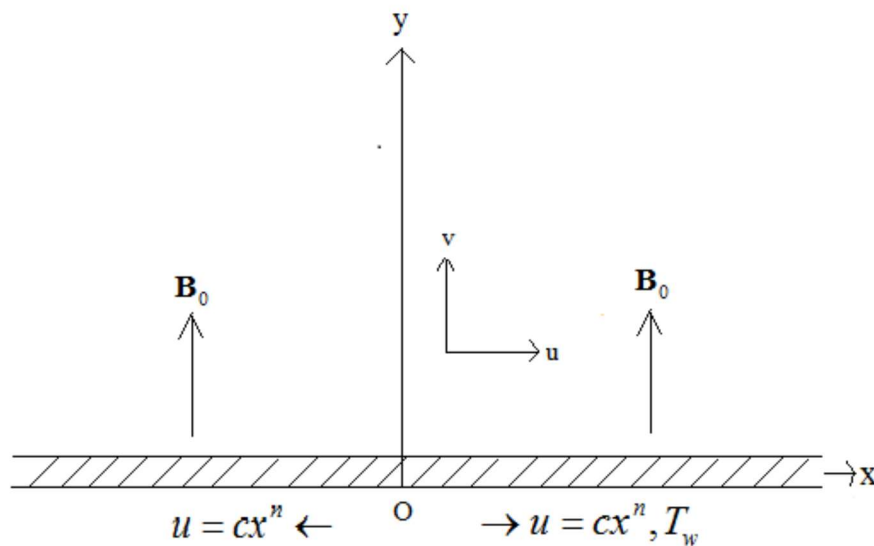
Bacteria, Algae and oxytans are some live examples of microorganisms as referred by Mahmud et al [24]. Topics such as aerodynamic extrusion of plastic sheet, material management conveyers were discussed by Vajravelu [25] and Char [27].

Glass blowing, paper production, drawing of plastic films and metal spinning are the applications referred by Buongiorno [26]. Viscous flow over nonlinear stretching sheet, being considered by Vajravelu et al [28]. Flow past a Rivlin-Ericksen fluid with MHD and mass transport phenomena were discussed by Vishalakshi et al [29]. An external magnetic field impacts the motion of electrically conducting fluid and on basis of this interaction, Lorentz force is being produced, Wakif et al [30]. EMHD is the main source to augment flow rates in micro channel, Sinha et al [31]. Investigation by Y S Daniel et al [32] involves Electro-MHD flow of nanofluid towards nonlinear stretching sheet with variable thickness in the presence of electric field. Viscous dissipation and Joule heating sturdily enhances the temperature field in the presence of magnetic field. The boundary layer equations of the fluid flow are the combination of momentum and heat transfer equations, which are formulated based on Maxwell's equation and OHM's Law in the presence of Electromagnetofluid dynamics (EMHD). The flow around a nonlinear stretching surface with variable thickness has been explored the impacts of magnetoelectric field, Joule heating, viscous dissipation, thermal radiation involving Cu/water as

nanofluid, Saiqa Sagheer et al [33]. Usharani et al [34] investigated the thermodynamic analysis of EMHD ternary nanofluid with shape factor effects over a shrinking sheet. EMHD has budding applications in radiation and viscous dissipation with heat transfer process. It is engaged in radio transmission across astronomy and geophysics to learn solar and stellar structures. Daniel et al [35] has explored that the velocity blockade is improved by electric parameter in their investigation over an expanding sheet with multislip and duality impacts. Areekera et al [36] have numerically clarified that the dynamics of EMHD convey of blood/gold nanoparticle substance over a nonlinear stretched surface involving Casson model with implications for its medical application. Most boundary layer flow problems encountered in the real world are nonsimilar, while various researchers have neglected this fact. So the author noted that the published literature has not examined the declaration of nanofluid flow along a stretching extended sheet using nonsimilar analysis with the consideration of EMHD effects. This research work finds application in industrial production and is useful to various researchers in this area of study. Neetu et al [37] considered the study of hybrid nanoparticles employing four factor rejoiner surface model to analyse EMHD flow across stretched sheet. Hamzeh et al [38] investigated the flow characteristics of Casson nano fluid predominantly considering its behavior under Darcy Forchheimer conditions precedent to a stretching sheet with convective boundary conditions which have noteworthy applications across various fields such as engineering and medical applications. This research work focuses on its relevance to cooling systems, material processing and bio medical engineering, where efficient flow and heat transfer characteristics are vital for the study of fluid

dynamics. Irfan et al [39] presented a computational technique for EMHD nano fluid flow establishing mathematical models for constant and variable fluid flows.

Provoked by the above studies, an exploration concerned to the effect of hydrodynamic and thermal slip on the act of fluid flow and thermal convey of Magneto hydrodynamic fluid over a porous upright nonlinear stretching sheet in the incidence of viscous and Ohmic dissipations, in porous media is well thought-out in this paper.



Schematic diagram of physical model for MHD flow past over a porous substrate attached to the nonlinearly stretching sheet.

### Mathematical Formulation of the problem

In the present work, a Viscous incompressible fluid flow and heat convey of an electrically conducting fluid past a spongy vertical nonlinear stretching sheet beneath the impact of transverse magnetic field, with hydrodynamic and thermal slip is considered



for investigation. An uniform transverse magnetic field of strength  $B_0$  is applied perpendicular to flow velocity. Consider a stretching sheet that pulls out of a slit, and is stretched, as in a polymer extrusion process. Here the speed at a point on the sheet, is proportional to the power of its distance from the slit. In deriving the following equations, it is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible. The surface of the sheet is insulated, therefore is governed by partial slip condition.

Further it is assumed that the sheet vary nonlinearly with the distance  $x$  from the leading edge, i.e.  $U_w = cx^n$

The Boussinesq approximation is engaged and the homogeneity and local thermal equilibrium in the porous medium are assumed. Hence in view of the conditions as mentioned above, and the usual boundary layer approximations, the governing boundary layer equations of motion, and heat transport, with buoyancy, viscous and Joules dissipation, hydrodynamic and thermal slip are presented as,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma}{\rho}(B_0^2 - EB_0)u - \frac{\nu}{k'}u - \frac{c_b}{\sqrt{k'}}u^2 \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\sigma B_0^2}{\rho c_p} \right) u^2, \quad (3)$$

and are governed by the following boundary conditions

$$u(x, y) = L \frac{\partial u}{\partial y} + cx^n, v = v_w(x), T = T_w + k_0 \frac{\partial T}{\partial y} \quad \text{at } y = 0,$$

$$u \rightarrow 0, T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty, \quad (4)$$

$$\text{Where } v_w(x) = -f_w \sqrt{\frac{\nu c(n+1)}{2}} x^{\frac{n-1}{2}}$$

Here  $u$  and  $v$  are the horizontal and transverse velocity components. Further,  $\mu$ ,  $\rho$ ,  $\alpha$ ,  $\beta$ ,  $T$ ,  $g$ ,  $c_b$ ,  $k'$  are the dynamic viscosity, fluid density, thermal diffusivity, thermal expansion coefficient, fluid temperature in the boundary layer, and acceleration due to gravity, drag coefficient and porosity respectively.

Introducing the following similarity transformation, i.e

$$\eta = \left( \frac{(n+1)c}{2\nu} \right)^{1/2} x^{\frac{n-1}{2}} y, u(x, y) = cx^n f'(\eta),$$

$$v(x, y) = - \left( \frac{(\nu c x)^{n-1}}{2} \right)^{\frac{1}{2}} \left( \frac{n-1}{2} \eta f'(\eta) + \frac{n+1}{2} f(\eta) \right) \quad (5)$$

$$\theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}. \quad (6)$$

The boundary layer flow, and heat transport equations, (2),(3), and (4), in view of the similarity transformation equations,(5) and (6) can be transformed into the following nonlinear ordinary differential equations.

$$f''' = ff'' - \left( \frac{2n}{n+1} \right) f'^2 - \left( \frac{2}{n+1} \right) \lambda \theta + M(f' - E_1) + N_1 f' + N_2 f'^2 \quad (7)$$

$$\theta'' = P_r f \theta' - \left( \frac{2n}{n+1} \right) f' \theta - Ec.Pr \left( f''^2 + M f'^2 \right) \quad (8)$$

In the same way, the boundary conditions (4) takes the form,

$$f(0) = f_w \quad f'(0) = 1 + \gamma f''(0), \quad \theta(0) = 1 + \delta \theta'(0), f'(\infty) = 0, \theta(\infty) = 0. \quad (9)$$

Where,

$$Ec = \frac{c^{\frac{5}{2}} x^{3n}}{\rho c_p}, \quad \gamma = L \sqrt{\text{Re}_x(1+n)}, \quad \text{Re}_x = \frac{c x^{n-1}}{\nu}, \quad M = \frac{c \sigma}{\rho(n+1)}$$

$$\lambda = \frac{Gr_x}{(\text{Re}_x)^2}, \text{ and } Gr_x = \frac{g \beta (T_w - T_\infty)}{\nu^2}, N_1 = \frac{x \nu (1-n)}{k' c}, N_2 = \frac{2c_b x}{\sqrt{k'(n+1)}}$$

$$\delta = k_0 \sqrt{\frac{(1+n)}{2}} \sqrt{\text{Re}_x} \quad \text{Where } B_0 = c x^{\frac{n-1}{2}}; E = a x^{\frac{n-1}{2}}; M = \frac{2\sigma c^2}{(n+1)c\rho};$$

$$E_1 = \frac{a}{c^2 U_w}$$

Where M-Magnetic Parameter,  $E_1$ =Electric Parameter

## NUMERICAL PROCEDURE

The highly nonlinear differential Eqs.(7) to(8), with boundary conditions (9) are solved numerically using Fourth-order RungeKutta shooting technique.

In the shooting procedure,guessing the values of  $f''(0)$  and  $\theta'(0)$  is very important. This method depends very much on our guesses. Numerical solutions are obtained for different values of the material parameters,i.e. magnetic parameter  $M$ ,

Electric Parameter  $E_1$ , stretching parameter  $n$ , Prandtl number  $Pr$ , slip parameter  $\gamma$ , thermal slip parameter  $\delta$ , Buoyancy parameter  $\lambda$ , Eckert number ( $Ec$ ), porous parameter  $N_1$ , Inertia Coefficient  $N_2$ , and suction/injection parameter  $f_w$ .

The step size is taken as,  $\Delta\eta = 0.01$  to have the convergence criterion to be  $10^{-6}$  in these cases. The maximum value value of  $\eta_{\infty}$  was taken for each iteration loop by  $\eta_{\infty} = \eta_{\infty} + \Delta\eta$ . The maximum value of  $\eta_{\infty}$  to each set of parameters is obtained when the value of the unknown boundary conditions at  $\eta = 0$  is not changed to successful loop with error less than  $10^{-6}$ .

## RESULTS AND DISCUSSION

In order to analyze the corporeal idea of the problem, the velocity, and temperature profiles are assigned various numerical values to the parameter, that occur in the problem i.e. numerical calculations were carried out for different values of Electric parameter  $E_1$  suction parameter  $f_w$ , magnetic parameter  $M$ , power law stretching parameter  $n$ , Prandtl number  $Pr$ , Eckert number  $Ec$ , buoyancy parameter  $\lambda$ , slip parameter  $\gamma$ , and thermal slip parameter  $\delta$ , their effects on flow and heat transfer characteristics are discussed graphically.

Fig 1 Shows the impact of electric parameter  $E_1$  on dimensionless velocity profile,  $f'(\eta)$  further it is observed that the velocity profile of the liquid adjacent to boundary layer predominantly enhanced, due to increase in the values of  $E_1$ .

Fig 2 shows the effect of magnetic parameter  $M$  on dimensionless velocity profile  $f'(\eta)$ , and it is noticed that velocity profile of the fluid in the boundary layer appreciably reduces with increase in values of  $M$ . This is because of the fact that, magnetic field introduces a retarding force which acts transverse to the direction of applied magnetic field. This force is Lorentz force, which decelerates the flow in the boundary layer,

resulting in thickening of momentum boundary layer and also it is noticed in increase of absolute value of velocity gradient at the surface of the sheet.

The effect of suction/injection parameter  $f_w$  ( $f_w < 0$ ),  $f_w$  ( $f_w > 0$ ), on horizontal velocity profiles are depicted in Fig3. It is noticed that the effect of suction parameter is to condense horizontal velocity in the boundary layer, where as in case of injection, i.e  $f_w$  ( $f_w > 0$ ), horizontal velocity increases. In case of large  $f_w$ , the boundary-layer assumptions do not admit a solution, where as it approaches the value unity resulting in boundary layer will almost be literally blown off at the surface of sheet, and the flow will be the same as that of stationery sheet.

Fig 4, shows the effect of suction/injection on dimensionless temperature profile. Here it is noticed that in case of increase of suction parameter( $f_w < 0$ ), temperature decreases, in the thermal boundary layer, and the thickness of boundary layer decreases, where as for increase of injection parameter( $f_w > 0$ ), there is decrease of thermal boundary layer thickness. Further it is observed that suction parameter ( $f_w < 0$ ) enhances, heat transfer coefficient much more than injection parameter ( $f_w > 0$ ). Thus, suction acts as a good means for cooling the surface than injection.

Figs 5 and 6, represents respectively the behaviors of the horizontal velocity and temperature profiles for different values of power law stretching parameter  $n$ , and here it is observed that increase in “ $m$ ” results in decrease of horizontal velocity profile which is more prominent for small values of  $n$ , where as temperature profile increases with the

increase of stretching parameter  $n$ . it is further noticed that, the sheet temperature variation has significant effect on thermal boundary layer. Further sheet temperature variation is observed to be more in the route of stretching rate.

fig 7,exhibits the effect of the Prandtl number on temperature profile. The temperature profile and thermal boundary layer thickness quickly decrease with increasing values of  $Pr$  as well as both slip and no slip conditions. Prandtl number acts as a means to increase fluid viscosity resulting in lessen in the flow velocity and temperature. Here thermal boundary layer thickness decreases with increasing Prandtl number, which is consistent with the findings of various reasearchers.

Next, let us pay attention,how velocity slip parameter  $\gamma$  effects the horizontal velocity profile.The velocity profile  $f'(\eta)$  for different values of the velocity slip parameter  $\gamma$  are shown in Fig 8. When there is slip, which means velocity is not zero at the surface of the sheet or the flow velocity adjacent to the sheet surface is not equal to the stretching sheet velocity.When  $\gamma$  increases,the fluid velocity increases. When the slip parameter  $\gamma$  increases in magnitude, more fluid is allowed to penetrate into the surface of stretching sheet. In the surrounding area of the sheet flow, gets accelerated. However, far away from the sheet the flow is deccelerated. Further it is noticed that, increasing the value of  $\gamma$  will increase the flow velocity, only because due to the slip condition, the pulling of the stretching sheet is partially transmitted into the fluid.

In fig 9, it is observed that,the effects of buoyancy parameter  $\lambda$  on horizontal velocity profile is shown graphically, and the effects of buoyancy force is found to be more effective for a lower Prandtl number fluid, which implies that lower Prandtl number

fluid is more susceptible to the effects of buoyancy force. Buoyancy is observed to deplete velocity in convective flows and therefore the reduced buoyancy force will manifest a boost in  $f'(\eta)$  as noticed in the figure.

It is noticed from Fig 10, that Eckert number controls the fluid motion in the boundary layer, and when  $Ec=0$ , is the case of no viscous dissipation, i.e for the case of ideal fluid, where its viscosity is negligible. The presence of  $Ec$  in the energy equation acts as a heat producing parameter due to the action of viscous stresses, where in dimensionless temperature overshoots at  $Ec=0.2$ . Further it is a well known fact that viscous dissipation is significant for those flows in which velocity gradient is large with fluids of high viscosity.

In Fig 11, the effects of thermal slip parameter  $\delta$  on temperature distribution is displayed. As the thermal slip increases, less heat is transferred from the sheet to the fluid and hence the temperature decreases, i.e increase in thermal slip parameter  $\delta$  provides the decreasing behavior on temperature profile.

**Fig.12:**Represents the effect of inertia coefficient  $N_2$  on velocity profile. From this we conclude that  $N_2$  enhances the thickness of momentum boundary layer.

Fig .13 represents the effect of porous parameter  $N_1$  over velocity profile. As Porous parameter increases, velocity decreases. Due to this, the velocity decreases in the boundary layer.

## CONCLUSION

The analysis of non darcy MHD boundary layer flow and heat transfer over a vertical stretching sheet with combined effect of hydrodynamic and thermal slip in a saturated



porous medium is studied. Numerical solutions for the governing boundary layer equations of motion is obtained, allowing the computation for flow and heat transfer characteristics for different values of porosity parameter, nonlinear stretching parameter, Prandtl number, Eckert number etc.

The results indicate that increasing hydrodynamic slip and electric parameter enhances velocity profile resulting in decrease of boundary layer thickness, whereas on the other hand increase of thermal slip reduces temperature in the thermal boundary layer region.

The velocity field is suppressed by increase in values of suction parameter resulting in enhancing skin friction coefficient, and the opposite results are noticed for injection.

The heat transfer rate at the surface of stretching sheet increases with increasing values of nonlinear stretching parameter, which results in increase of thermal boundary layer thickness.

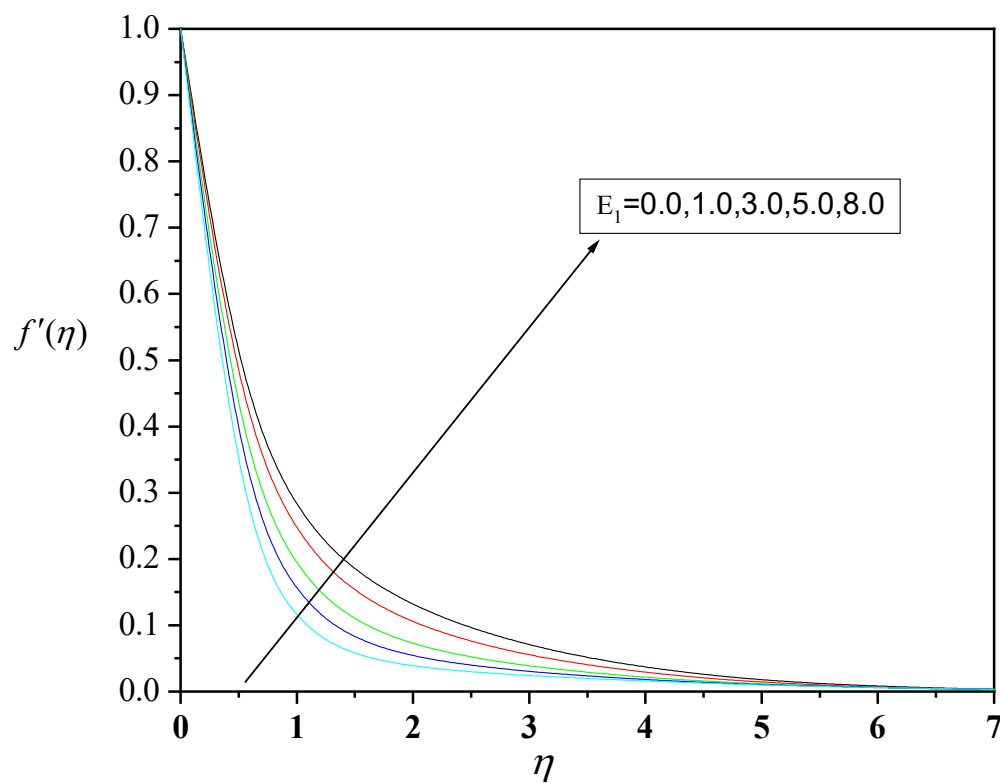
## References

- [1] Fischer EG. Extrusion of plastics. New York: Wiley; 1976.
- [2] Sakiadis BC. Boundary layer behaviour on continuous solid surfaces: I boundary layer on a continuous flat surface. *AIChE J* (1961) 221–225.
- [3] Crane L. Flow past a stretching plate. *Z Angew Math Phys* 21(1970)645–647.
- [4] H.S. Takhar, M.A. Ali, A.S. Gupta, Stability of magnetohydrodynamic flow over a stretching sheet, in: Lielpeteris, Moreau (Eds.), *Liquid Metal Hydrodynamics*, Kluwer Academic Publishers, Dordrecht, 1989, pp. 465–471.
- [6] M. Kumari, H.S. Takhar, G. Nath, MHD flow and heat transfer over a stretching surface with prescribed wall temperature or heat flux, *Thermo-Fluid Dynamics. Wärme und Stoffübertragung*. 25 (1990) 331–336.
- [5] H.I. Andersson, MHD flow of a viscous fluid past a stretching surface, *Acta Mech*. 95 (1992) 227–230.

- [7] T. Watanabe, I. Pop, Hall effects on magnetohydrodynamic boundary layer flow over a continuous moving flat plate, *Acta Mech.* 108 (1995) 35–47.
- [8] I. Pop, T.Y. Na, A note on MHD flow over a stretching permeable surface, *Mech. Res. Comm.* 25 (3) (1998) 263–269.
- [9] N. Chaturvedi, On MHD flow past an infinite porous plate with variable suction, *Energy Convers. Manage.* 37 (5) (1996) 623–627.
- [10] G.A. Rao, A.S.S. Murthy, V.V.R. Rao, Hydrodynamic flow and heat transfer in a saturated porous medium between two parallel porous walls in a rotating system, in: *Proceedings of the Eighth National heat and mass transfer conference*, AU College of Engineering Vizag, 1985.
- [11] M. Turkyilmazoglu, Heat and mass transfer of the mixed hydrodynamic/thermal slip MHD viscous flow over a stretching sheet, *Int.J.Mech.Sci.* 53(2011)886-896.
- [12] Samir Kumar Nandy and Tapas Ray Mahapatra, Effects of slip and heat generation /absorption on MHD stagnation flow of nanofluid past a stretching /shrinking surface with convective boundary conditions, *Int.J.Heat Mass Transfer*,64(2013)1091-1100.
- [13] Turkyilmazoglu, Exact analytical solution for heat and mass transfer of MHD slip flow in nanofluids, *Chem. Engng. Sci.* 84 (2012) 182–187.
- [14] E.H.Aly and K.Vajravelu, Exact and numerical solutions of MHD nano boundary layer flows over stretching surfaces in a porous medium, *Appl.Math.Comp.*,232(2014)191-2014.
- [15] Bikash Sahoo and Sebastian Poncet, Flow and heat transfer of a third grade fluid past an exponentially stretching sheet with partial slip boundary condition, *Int.J.Heat Mass Transfer*,54 (2011) 5010–5019.
- [16] Alin V Rosca and Ioan Pop, Flow and heat transfer over a vertical permeable stretching/shrinking sheet with a second order slip, *Int.J.Heat Mass transfer*,60 (2013) 355–364.
- [17] Liancun Zhenget al, Flow and radiation heat transfer of a nano fluid over a stretching sheet with velocity slip and temperature jump in porous medium, *J. Franklin Institute* 350 (2013) 990–1007.
- [18] M. Turkyilmazoglu, Heat and mass transfer of MHD second order slip flow *Computers & Fluids* 71 (2013) 426–434.
- [19] Turkyilmazoglu M. Analytic heat and mass transfer of the mixed hydrodynamic/thermal slip MHD viscous flow over a stretching sheet. *Int J Mech Sci*,53 (2011)886–96.

- [20] Turkyilmazoglu M. Multiple solutions of heat and mass transfer of MHD slip flow for the viscoelastic fluid over a stretching sheet. *Int J Therm Sci*,50(2011)2264–2276.
- [21] Fang TG, Yao S, Zhang J, Aziz A. Viscous flow over a shrinking sheet with a second order slip flow model. *Commun Nonlinear Sci Numer Simulat*15 (2010)1831-1842.
- [22] Nandeppanavar MM, Vajravelu K, Abel MS, Siddalingappa MN. Second order slip flow and heat transfer over a stretching sheet with non-linear Navier boundary condition. *Int J Therm Sci*,58 ( 2012)143–150.
- [23] W.A. Khan, I. Pop, Boundary-layer flow of a nanofluid past a stretching sheet, *Int. J. Heat Mass Transfer* 53 (2010) 2477–2483.
- [24]Mahmood K, Rana S, Al-Zubaidi a,Mehmood R,Saleem S, interaction of Lorentz force with cross swimming microbes in couple stress fluid past a porous riga plate, *Int.Commun.Heat Mass Trans* (2022)(138) 1-12.
- [25]Vajravelu.K, Viscous flow over a nonlinear stretching sheet, *App.Math.Comp*, (2001)(15)(3)281-288.
- [26]Buongiorno.J, Convective transport of nanofluids,*J.Heat Transfer* (2006)(128)(3)240-250.
- [27]Char M I ,Heat transfer of a continuoos stretching surface with suction and blowing,*J.Math.Anal.Appl*(1998)(135)(2)568-580.
- [28]Vajravelu.K, Viscous flow over a nonlinear stretching sheet,*Appl.Math.Comput* (2001)(124)(3)281-288.
- [29] vishalakshi A B ,Mahabaleshwar U S ,Sheikknejad Y, Impact of mass transportation on Rivlin ericksen liquid flow over a stretching sheet in porous media with thermal communication, *Trans.Porous media*(2022)(142)353-381.
- [30]Wakif ali C,Animasunni II, Zaydan M Hassan, W Schaqui R Novel, physical insights into the thermodynamic irreversibilities with dissipativeEMHD fluid flows over a moving riga plate in the existence of wall suction and joule heating effects; A comprehensive numerical investigation, *Arab.J.Sci.Eng*(2020)(45)(11)9423-9438.
- [31]Sinha a ,Shit G C,Electromagnetic hydrodynamic flow of blood and heat transfer in a capillary with thermal radiation, *J.Magnetism.Magn.Mater*(2015)(378)143-151.
- [32]Yahaya Shagaiya Daniel,Zainal Abdul Aziz,Zuhailla Ismail,Faisal Shah, impact of thermal radiation on electrical MHD flow of nanofluid over a nonlinear stretching sheet with variable properties, *Alexandria.eng.J*(2018)(57)2187-2197.
- [33]Saiqha Sageer,Raheeda razzaq and Kambiz Vafai, Local nonsimilar solutions for EMHD nanofluid flow with radiation and variable heat flux along stretching sheet, *numerical .heat.transfer. Part A*(2024)(22)3923-3940.
- [34] Usharani etal, Thermodynamic analysis of EMHD ternary nanofluid with shapefactor effects over a shrinking sheet: a nonfourier heat flux model, *Numer.Heat.transfer.Part A Appl*(2024)(85)367-382.

- [35]Daniel Y S,Z A Aziz,Z Ismail,and A Bahar, unsteady EMHD dual stratified flow of nanofluid with slip impacts,Alexandria.Eng.J(2020)(59)(1)177-189.
- [36] s Areekera, A S Sabu, and A Mathew,K S Parvathy,and P S Rana, Significance of nanoparticle radius on EMHD casson blood-Gold nanomaterial flow with nonuniform heat source and Arrhenius kinetic,J.Therm.Anal.Calorim.(2023)(148)(17)8945-8968.
- [37] Neet T S, S Areekera, A S Sabu, A Mathew, and K K Anakha,Bioconvective EMHD hybrid nanoliquid flow over a stretching sheet with stratification effects,a four factor response surface optimized model, Waves.Random.Complex.Media(2022)(32)1-26.
- [38]Hamzeh taha,Alkasasbeh,Modelling the flow of casson nanofluid on a stretching sheet with heat transfer;A study of electric MHD and Darcy Forchheimer effects, Partial.DiffEqns.Appl.Maths(2025)(13)10-12.
- [39]Irfan M, Farooq M A,Iqra J,A new computational technique design foer EMHD nanofluid flow over a variable thickness surface with variable liquid charecteristics, Front.Phy(2020)(8)66-



**Fig 1 Velocity profile  $f'(\eta)$  versus similarity variable  $\eta$  for various values of  $E_1$  when,  $M=3.0$ ,  $\lambda=3.0$ ,  $f_w=1.0$ ,  $n=0.7$ ,  $\gamma=2.0$ ,  $N_1=0.2$ ,  $N_2=0.1$ .**

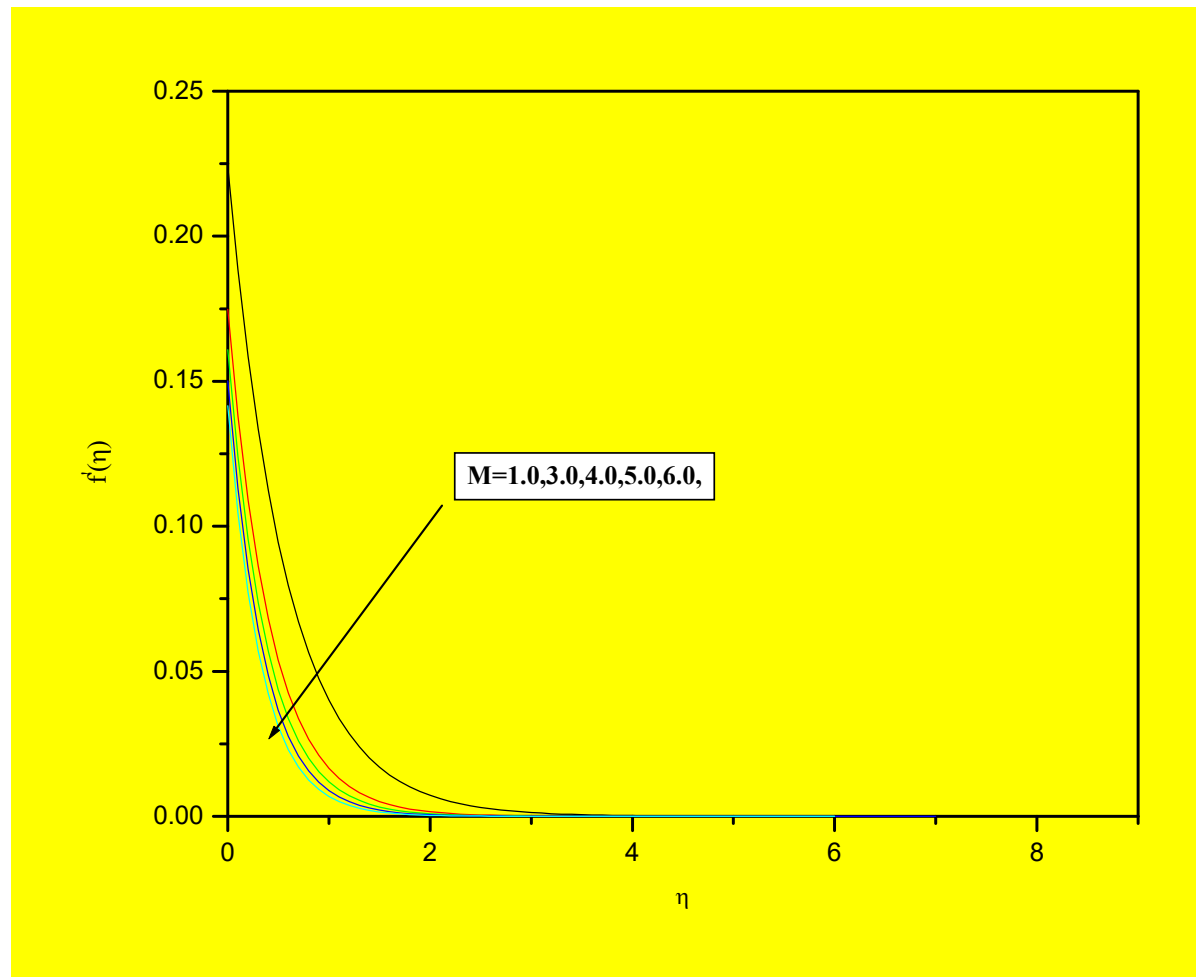


Fig 2. Velocity profile  $f'(\eta)$  versus similarity variable  $\eta$  for various values of  $M$  when  $\lambda=3.0, f_w=1.0, n=0.7, \gamma=2.0, E_1=0.1, N_1=0.2, N_2=0.1$ .

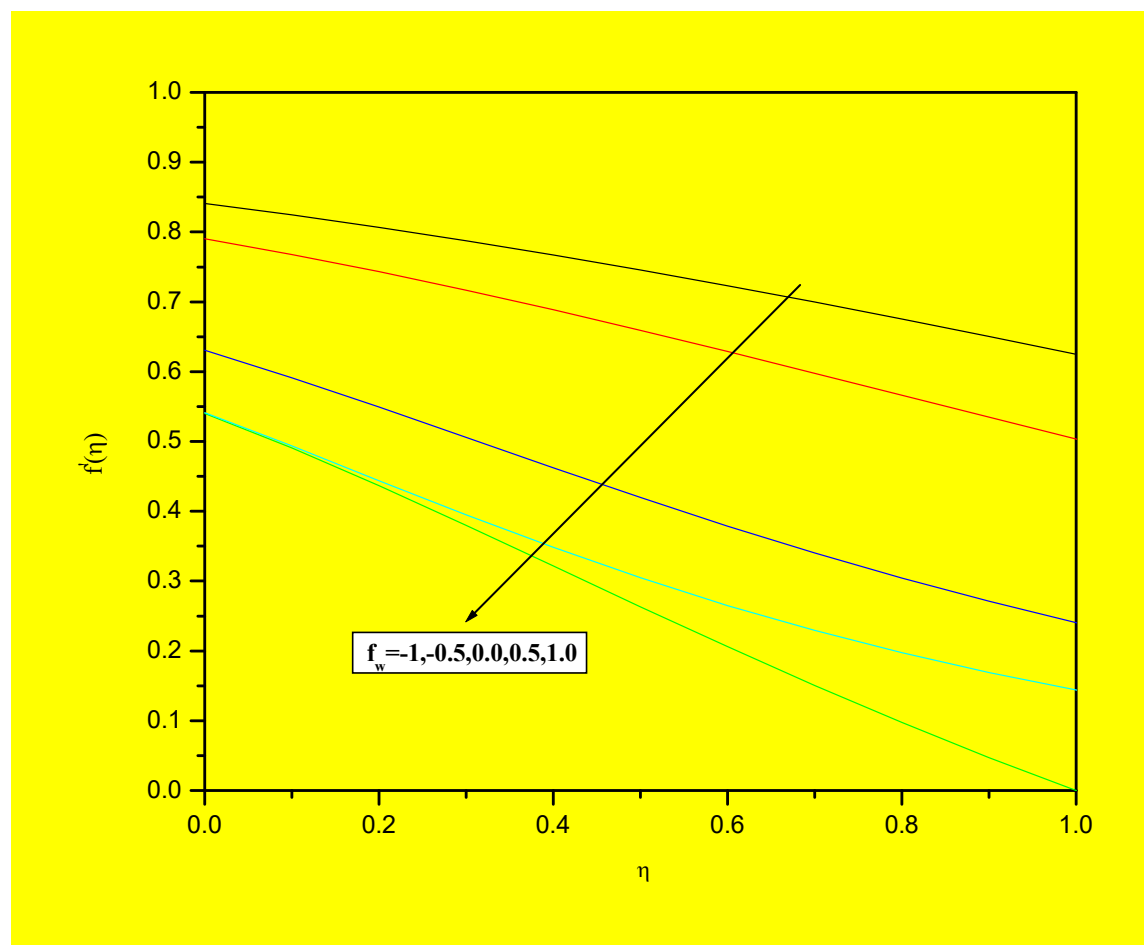


Fig 3 Velocity profile  $f'$  versus similarity variable  $\eta$  for various values of  $f_w$

When,  $E_1=0.1, \lambda=3.0, n=0.7, M=2.0, Y=2.0, N_1=0.1, N_2=0.1$ .

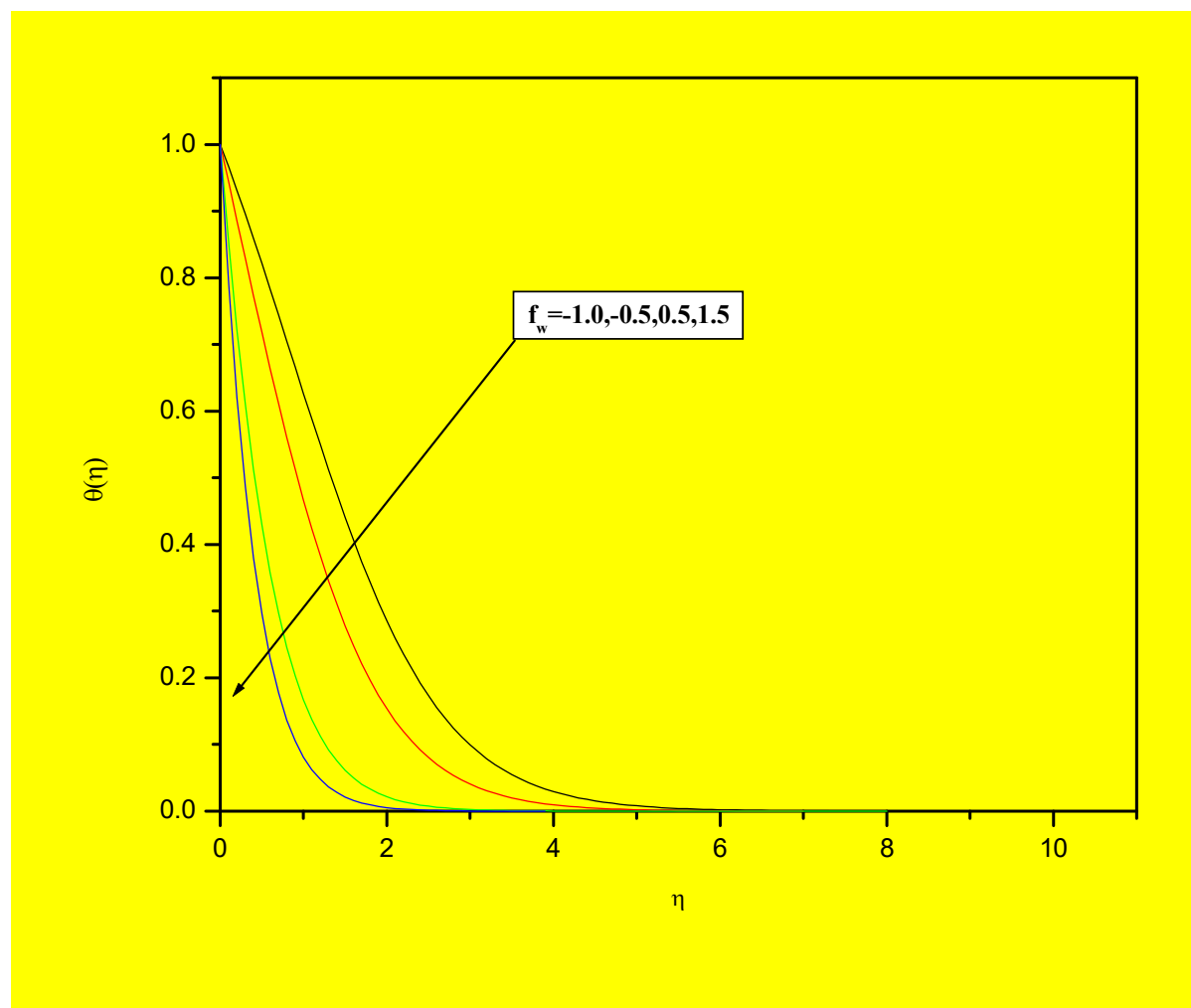


Fig 4 Temperature profile  $\theta(\eta)$  versus similarity variable  $\eta$  for various values of  $f_w$

when  $n = 3.0, Pr = 2.0, \gamma = 2.0, Ec = 2.0$ .



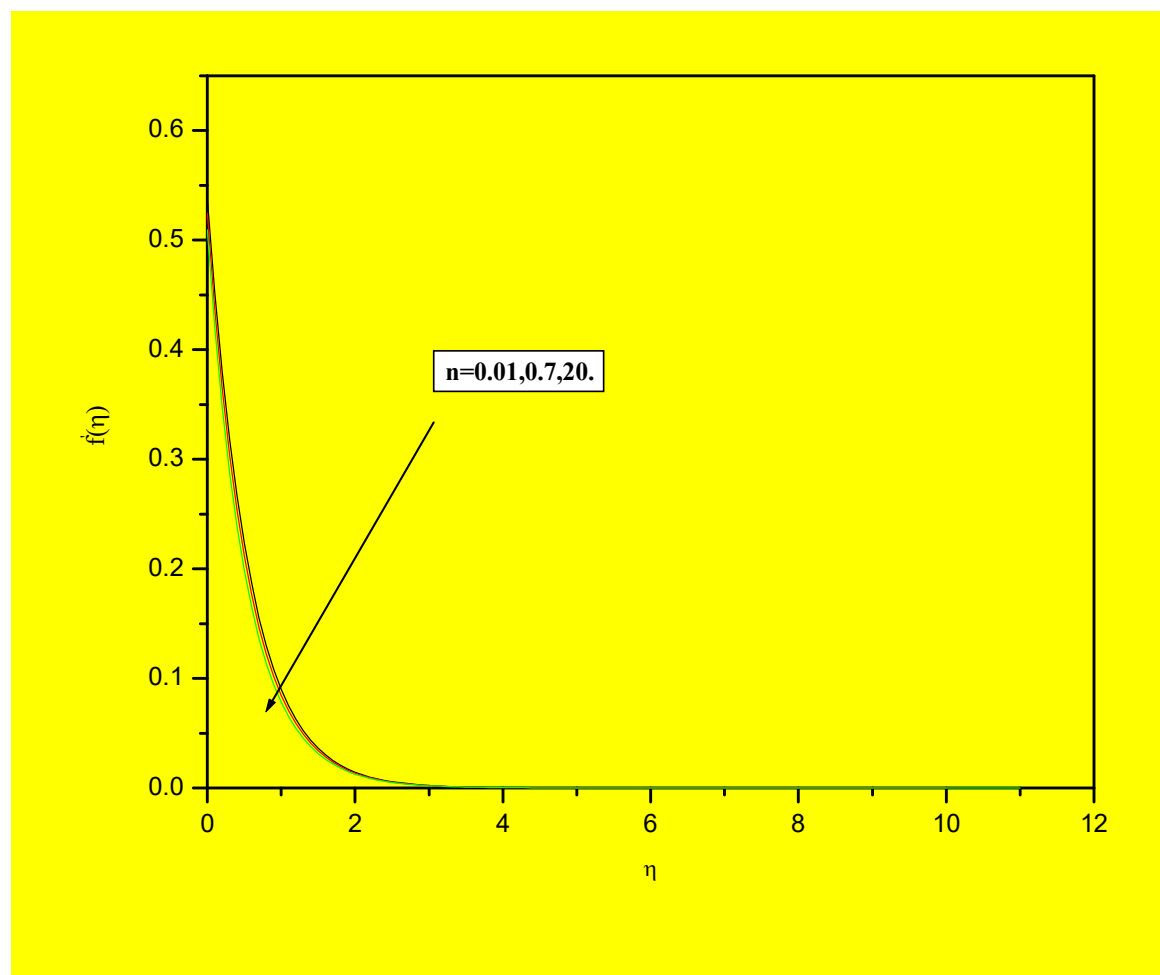


Fig 5 Velocity profile  $f'(\eta)$  versus  $\eta$  for various values of  $n$ , when

$$M = 3.0, \lambda = 2.0, \gamma = 5.0, f_w = 2.0, N_1 = 0.2, N_2 = 0.1.$$

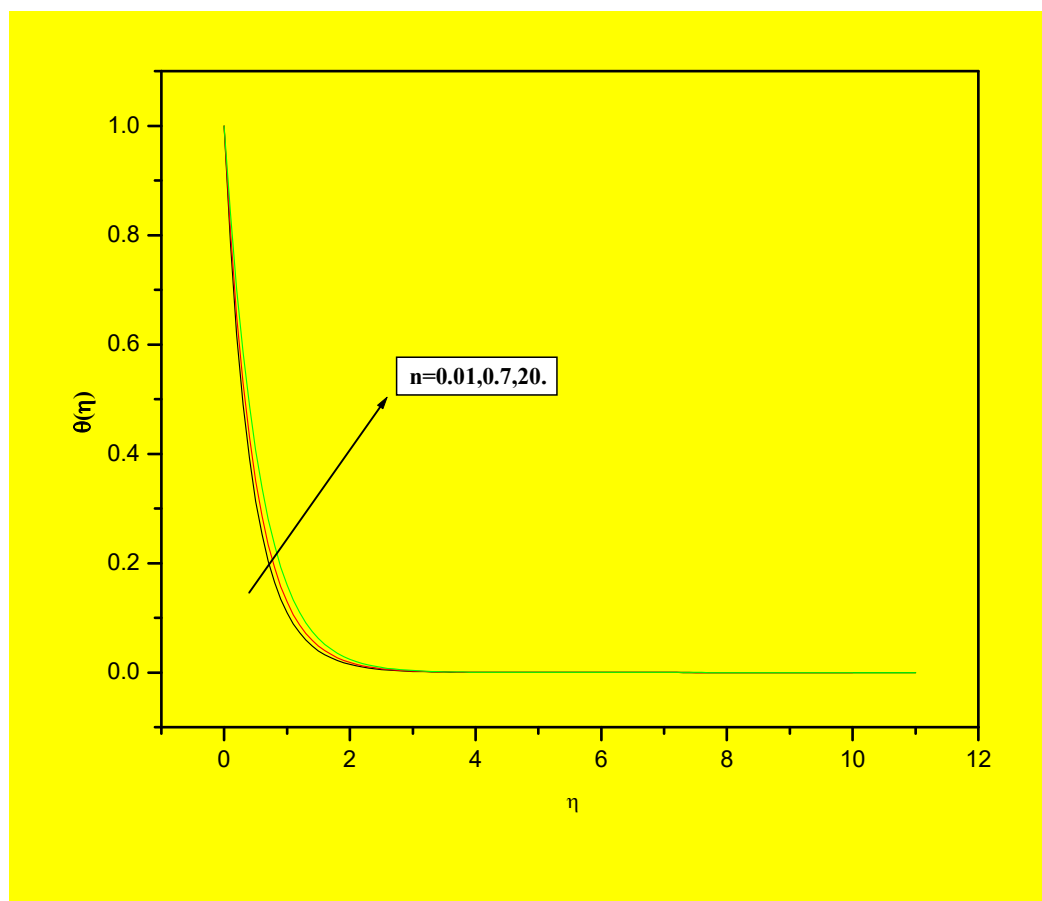


Fig 6 Temperature profile  $\theta(\eta)$  versus  $\eta$  for various values of  $n$ , when  $Pr=1.5$ ,  $Ec=2.0$ ,  $f_w=1.0$ ,

$\gamma=0.5$ ,

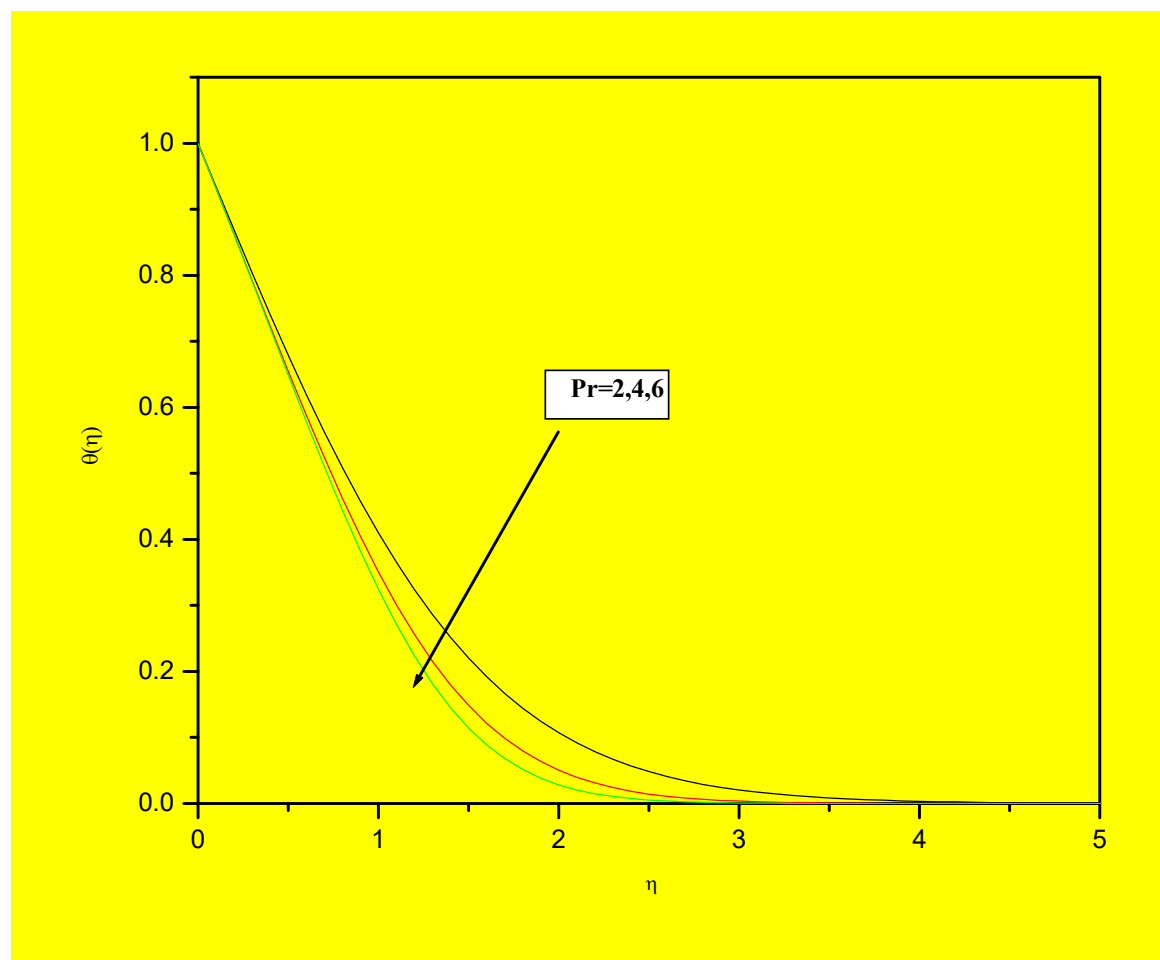


Fig 7 Temperature profile  $\theta(\eta)$  versus similarity variable  $\eta$  for various values of Pr, when

$n=2.0$ ,  $Ec=1.0$ ,  $f_w=1.0$ ,  $\mathcal{V}=1.0$ .

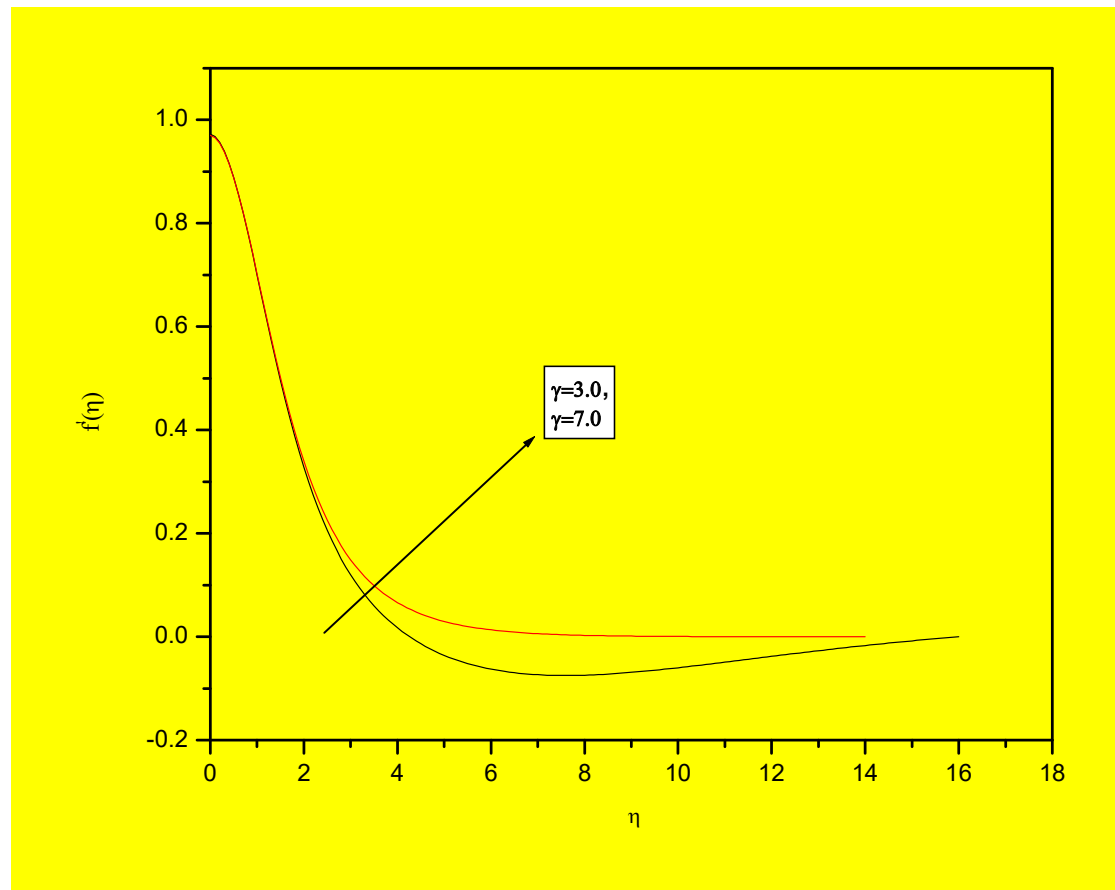


Fig 8 Velocity profile  $f'(\eta)$  versus similarity variable  $\eta$  for various values of

$n=2.0, M=2.0, \lambda=3.0, f_w=-1.0, N_1=0.2, N_2=0.1.$

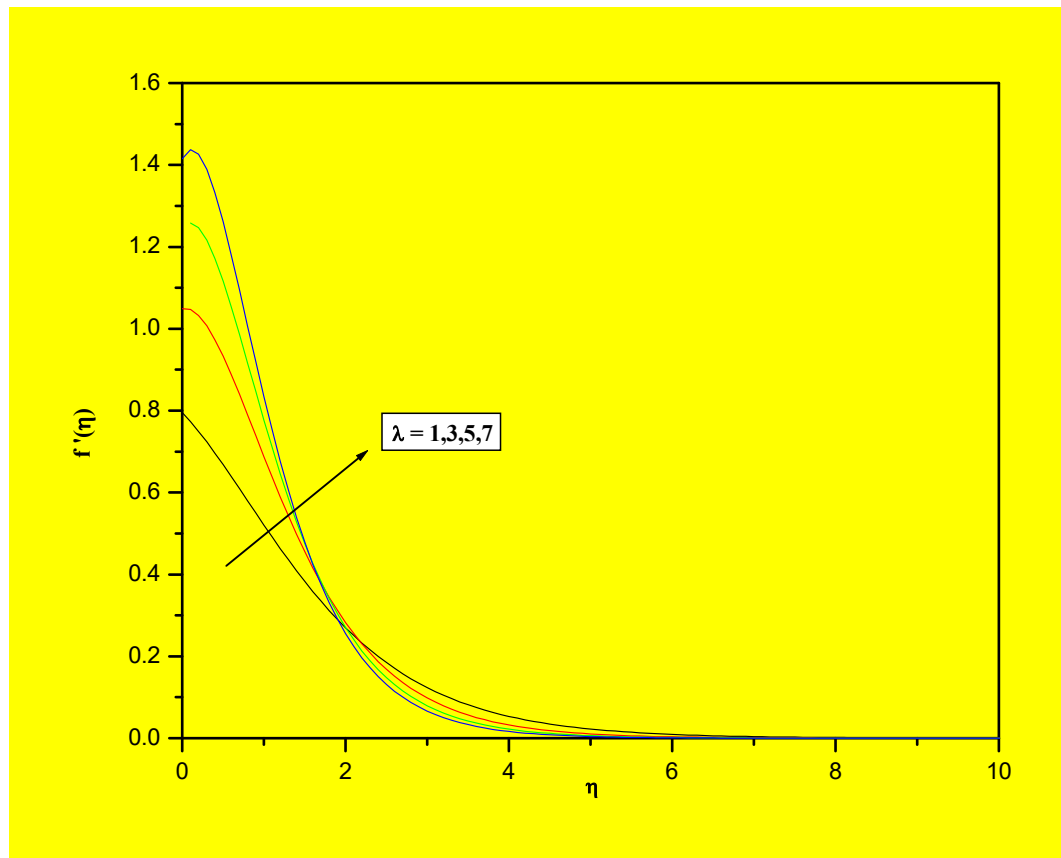


Fig 9, Velocity profile  $f'(\eta)$  versus similarity variable  $\eta$  for various values of buoyancy parameter  $\lambda$ , when  $n=2.0, M=2.0, f_w=-0.5, \gamma=1.0, N_1=0.2, N_2=0.1$ .

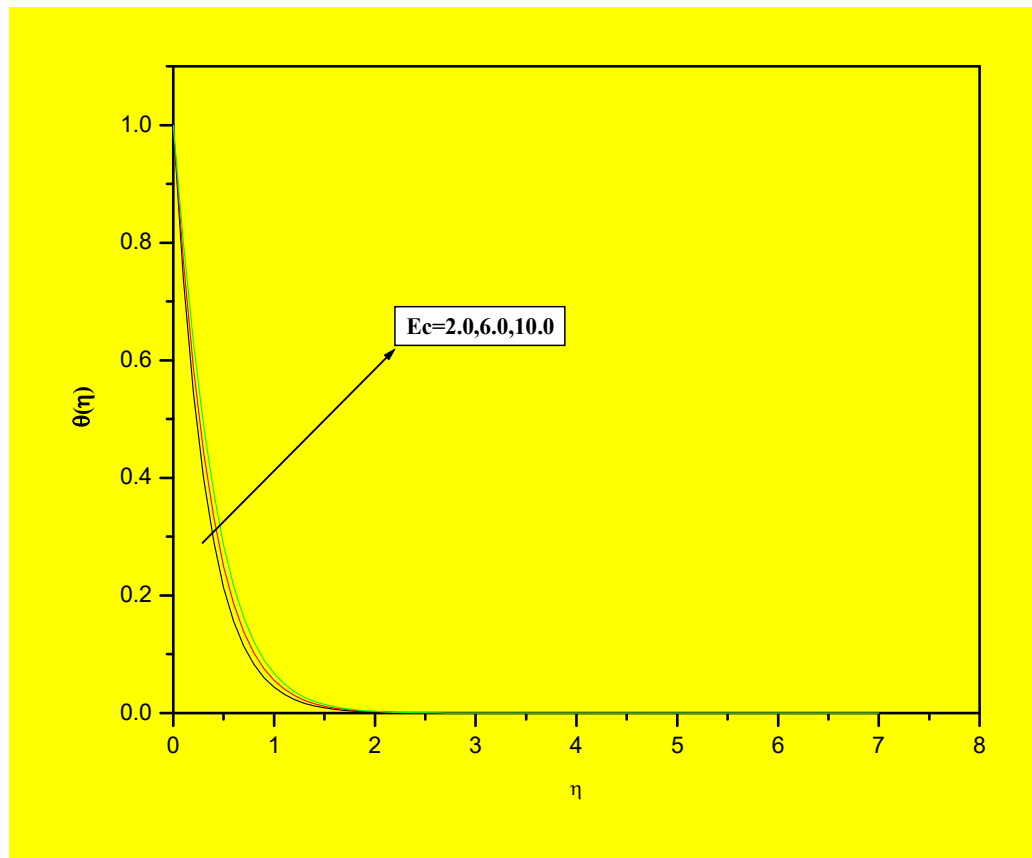


Fig 10 ,Temperature profile  $\theta(\eta)$  versus similarity variable  $\eta$  for various values of  $Ec$ ,

When  $n=2.0, Pr=3.0, f_w=1.0, \delta=2.0$ .

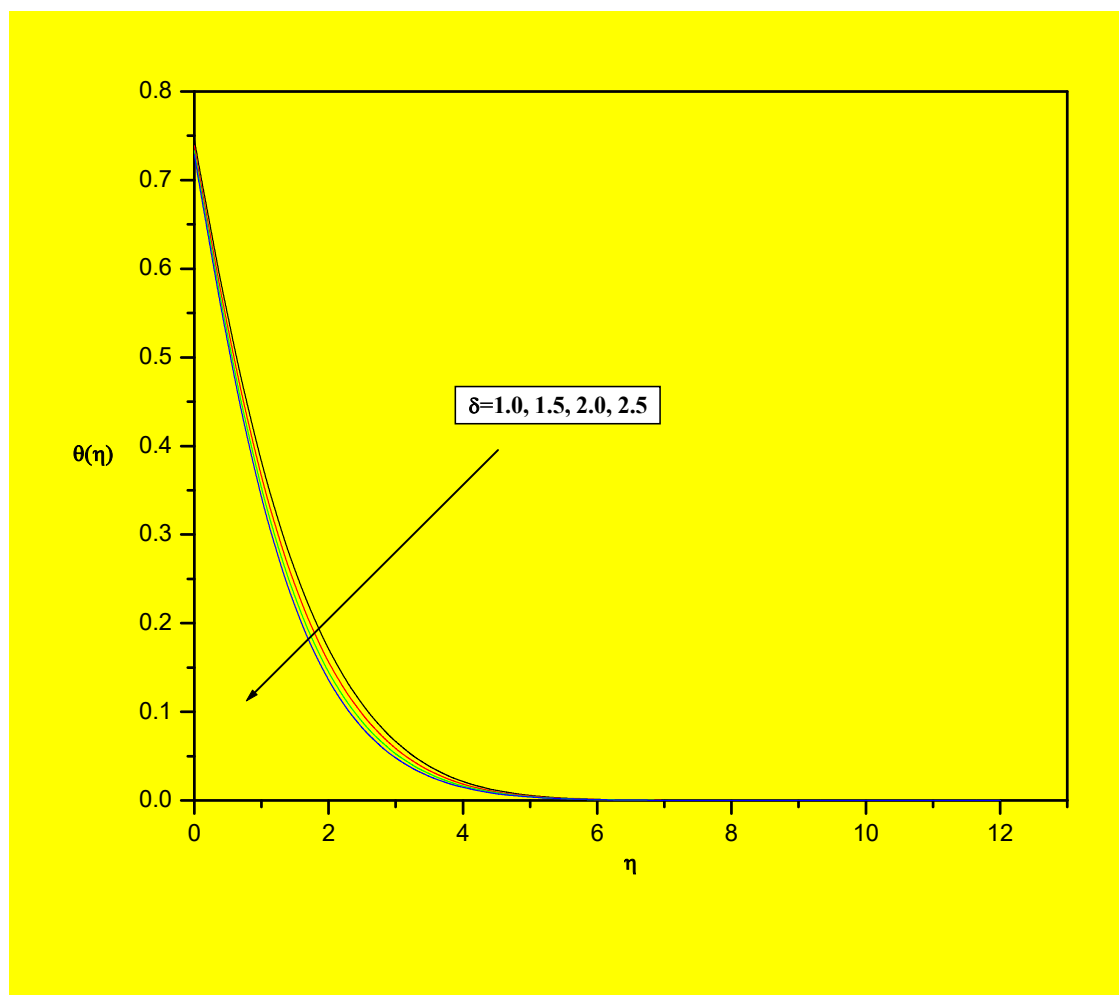


Fig 11, Temperature profile  $\theta(\eta)$  versus similarity variable  $\eta$  for various values of  $\delta$ ,

When  $Ec=2.5, \lambda=3.0, Pr=1.5, \gamma=0.5$ .

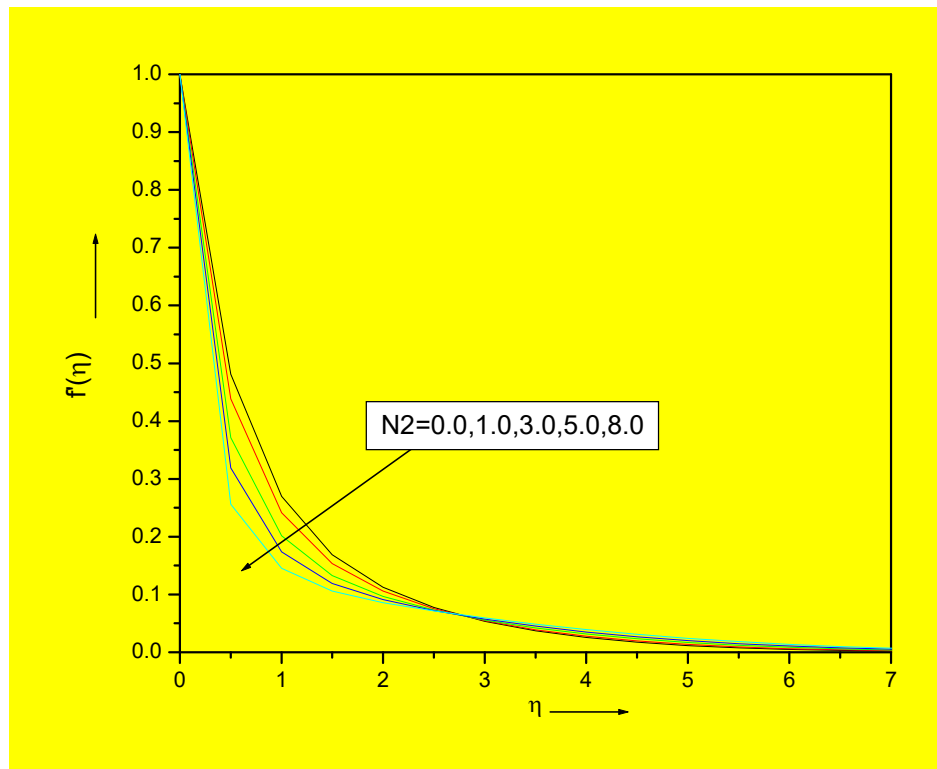


Fig 12, Velocity profile  $f'(\eta)$  versus similarity variable  $\eta$  for various values of

inertia coefficient  $N_2$ , when  $n=2.0, N_1=2.0, f_w=-0.5, \gamma=1.0, \lambda=2.0$ .



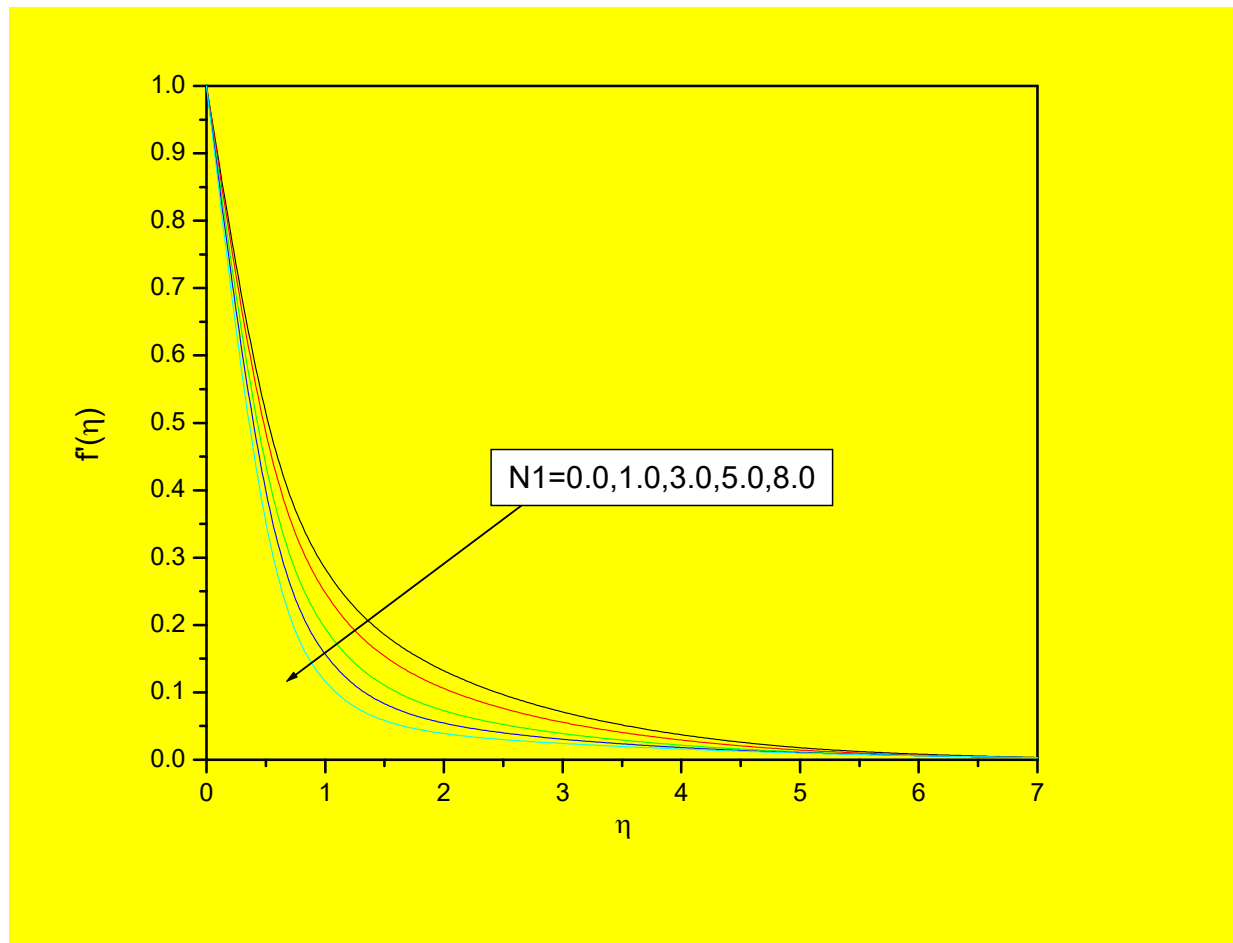


Fig 13 Velocity profile  $f'(\eta)$  versus similarity variable  $\eta$  for various values of porous parameter

$N_1$  when,  $n=2.0, M=2.0, \lambda=3.0, f_w=-1.0, N_2=1.0$ .