Higher-Degree Polynomial Regression Improves Breast Cancer Classification: Evidence from Feature Selection and False Negative Reduction

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Abstract: Breast cancer (BC), a malignant proliferation of breast tissue cells, remains a leading global health concern despite advances in early detection. While family history and known risk factors (RFs) contribute to BC, many cases occur without identifiable RFs due to complex interrelationships among variables. Early detection and accurate risk assessment remain critical challenges. This study leverages polynomial regression (PR) models—quadratic (QDPR), cubic (CIPR), and quartic (QRPR)—to analyze non-linear relationships between clinicopathological features and BC outcomes using the Wisconsin Breast Cancer Dataset (WBCD). Our approach achieves zero false negatives, a crucial advancement for clinical diagnosis, and identifies key risk factors through iterative feature selection. Comparative results demonstrate that QRPR outperforms lower-degree models, with 96.2% accuracy and superior r2-score (0.835), highlighting its potential to enhance BC prediction and inform personalized treatment strategies.

Keywords: Breast Cancer, Complex Interrelationship, Wisconsin Breast Cancer Dataset, False Negatives, Polynomial Regression, Quadratic Polynomial Regression, Cubic Polynomial Regression, Risk Factors ,Quartic Polynomial Regression

1. Introduction

Breast cancer (BC) is the disease that caused by genetic mutations, which leads to malignant tumors in breast tissue, primarily affecting connective tissue, lobules, or milk ducts [1,2]. Clinical manifestations which include palpable lumps, skin dimpling, breast pain, and abnormalities of the nipples [3]. By addressing these issues and non-linear effects of risk factors, including age, genetics, and lifestyle (e.g., alcohol consumption, obesity), which are well-documented [4], this study closes a significant gap in the literature.

For modeling non-linear relationships in BC research, polynomial regression (PR) has become a potent tool. while prior studies used quadratic (QDPR) and cubic (CIPR) models to assess risk factors like breast density and BMI [5,6], these lower-degree polynomials often fail to capture complex feature interactions. Recent work with quartic regression (QRPR) shows promise in tumor behaviour prediction [8], but no systematic comparison of PR degrees exists for BC classification. Our study bridges this gap by rigorously evaluating QDPR, CIPR, and QRPR on the Wisconsin Breast Cancer Dataset (WBCD), demonstrating that higher-degree polynomials (QRPR) improve accuracy while maintaining zero false negatives—a critical advance for clinical deployment. We validate our approach using the Wisconsin Breast Cancer

Dataset (WBCD), a benchmark dataset containing 569 cases with 30 tumor morphology features [20]. Beyond oncology, PR's versatility in modelling non-linear trends (e.g., in finance [10] and traffic flow [11]) underscores its suitability for BC risk prediction. The paper is organized as follows: Section 2 details PR theory; Section 3 formalizes the problem; Sections 4–6 present methodology, experiments, and results; and Section 7 discusses clinical implications.

2. Polynomial Regression

2.1 About Polynomial Regression

Polynomial regression is a statistical method used to model nonlinear relationships between one or more independent variables and a dependent variable by fitting a degree-n polynomial equation to the data. Unlike simple linear regression, which assumes a straight-line relationship, polynomial regression captures curvilinear trends through higher-order terms (e.g., quadratic, cubic) [12]. The highest power of independent variable(s) in polynomial equation are linear (1), quadratic (2), cubic (3), quartic polynomial or fourth-degree polynomial (4).

This technique is widely applied in fields requiring flexible modelling of complex patterns, including oncology (e.g., breast cancer risk prediction [6–8]), time series forecasting [9], and traffic flow analysis [11]. Its adaptability makes it particularly useful when relationships between variables are nonlinear but deterministic [13].

2.2 Quadratic Polynomial Regression (Degree 2)

QDPR is a type of regression where the relationship between independent variable and dependent variable is modelled as quadratic polynomial equation of the form:

$$Y= ax^2+bx+c \quad (1)$$

Where 'x' variable is an independent variable, 'y' variable is dependent variable and a, b, c are co-efficient, QDPR is used to model non-linear relationships.

2.3 Cubic Polynomial Regression (Degree 3)

CIPR regression analysis that involves the parameterization of indirect effects, enabling the examination of complex relationships between dependent variables and independent variable is modelled as cubic polynomial equation of the form:

$$Y= ax^3+bx^2+cx+d+€ \quad (2)$$

Where 'x' variable is independent variable, 'y' variable is dependent variable and a, b, c, d are co-efficient and € is the error term, CIPR is used to model more complex non-linear relationships.

2.4 Quartic Polynomial Regression (Degree 4)

QRPR is a regression technique that examines the relationships between dependent variable and a independent variable, is modelled as fourth-degree polynomial equation of the form:

$$Y= ax^4+bx^3+cx^2+dx+e+€ \quad (3)$$

Where 'x' variable is independent variable, 'y' variable is dependent variable and a, b, c, d, e are co-efficient and € is the error term, QRPR is used to model more complex non-linear relationship than CIPR.

3. Problem Statement

Breast cancer remains a leading cause of cancer-related mortality in women worldwide, with early and accurate diagnosis being critical for improving survival rates. While machine learning has been widely applied to breast cancer prediction, most studies focus on linear models (e.g., logistic regression) or complex black-box algorithms (e.g., deep learning), often overlooking the potential of polynomial regression (PR) to model non-linear relationships between clinicopathological features and disease outcomes [1,2].

This paper addresses three key gaps:

1. Limited comparative analysis: Prior works [3,4] have not systematically evaluated which PR variant (quadratic/QDPR, cubic/CIPR, or quartic/QRPR) best captures non-linear risk-factor interactions in breast cancer.
2. Clinical interpretability: Complex models like neural networks sacrifice interpretability, while PR offers transparent coefficients for clinical decision-making [5].
3. False-negative minimization: No existing study has demonstrated PR's ability to achieve zero false negatives—a critical requirement for clinical deployment [6].

We aim to:

1. Rigorously compare QDPR, CIPR, and QRPR on the Wisconsin Breast Cancer Dataset (WBCD) to identify the optimal degree for accuracy and interpretability.
2. Quantify performance gains over traditional linear models (e.g., 96.2% accuracy with QRPR vs. 92% with logistic regression in [7]).
3. Propose a clinically actionable framework that balances predictive power with interpretability for oncologists.

This work contributes to both ML research (as the first systematic study of high-degree PR in breast cancer) and clinical practice (through false-negative reduction and feature importance analysis).

4. Methodology

4.1 Dataset and Preprocessing

The Wisconsin Breast Cancer Dataset (WBCD) was utilized, comprising 569 instances with 30 real-valued features describing tumor characteristics (e.g., radius, texture, concavity) and binary labels (benign/malignant). The preprocessing pipeline included:

a. Data Cleaning: Identification and imputation of missing values (NAN) using median substitution.
b. Normalization: Feature scaling via standardization (Z-score normalization) to ensure equal contribution of all variables.
c. Train-Test Split: Partitioning into training (75–80%) and test sets (20–25%) with stratified sampling to preserve class distribution.

4.2 Model Development and Evaluation

Three polynomial regression (PR) variants were implemented:

a. Quadratic (QDPR): as given in Equation (1)
b. Cubic (CIPR): as given in Equation (2)
c. Quartic (QRPR): as given in Equation (3)

The result section shows prediction and accuracy of the classification of benign type cancer and malignant type cancer of applied variants, Data analytics is performed on variants of Polynomial Regression models to evaluate the problem statement stated in section 3.

5. About Dataset

5.1 Dataset Overview:

The Wisconsin Breast Cancer Database (WBCD) is a widely used dataset collected by from Kaggle.com [14]. It contains 569 samples, each described by 32 attributes (30 real-valued features and 2 classes: benign tumors (357 cases, 62.9%) and malignant tumors (212 cases, 37%). The features include measurements like radius, texture, and smoothness, categorized into mean, standard error, and "worst" values. The data is stored in CSV format, making it accessible for machine learning tasks.

5.2 Scaling:

To ensure fair comparison across features, the dataset is normalized using the Standard Scaler from scikit-learn. Normalization adjusts the feature scales to prevent variables with larger ranges from overshadowing others, improving model accuracy and training efficiency. The Standard Scaler transforms each feature using the formula $Z=(X-\mu)/\sigma$, where X is the original value, μ is the mean, and σ is the standard deviation. This process reduces outliers, noise, and inter-feature correlations, resulting in a standardized dataset suitable for machine learning algorithms.

6. Experimental Setup

6.1 Computational Environment

The hardware requirements for the experiments are listed in Table 1, and the experiments were carried out with Anaconda, a complete open-source data science platform that guarantees consistency across operating systems (Windows, macOS, and Linux).

Table 1: System Specifications for Experimentation

Si.No	Component	Specification
1	Operating System	Windows10, 64bit OS
2	Processor	Intel core-i5, GPU, 11^{th} Generation
3	RAM	8GB
4	Storage	1TB, SSD

6.2 Tools and Libraries

- **Jupyter Notebook**: is an interactive web-based environment for data exploration and sharing documents with live code, visualizations, and text.
- **Python**: Primary programming language for data preprocessing, machine learning, and statistical analysis.
- **Scikit-Learn**: Used for implementing machine learning algorithms, including Polynomial Regression variants.
- **Matplotlib**: Employed for data visualization and performance metric analysis.

6.3 Model Validation and Evaluation

To ensure robustness, 10-fold cross-validation was applied, where the dataset was partitioned into 10 subsets (folds). Each fold was used once as the test set, and the average performance across all iterations was computed to mitigate overfitting.

Performance Metrics:

The following metrics were used to evaluate model performance (Table 2):

Table 2: Evaluation Metrics and Formulas

Metrics	Formula
F1-Score	= 2*(P*R)/(P+R)

Accuracy	=((tp+tn) /(tp+tn+fp+fn))*100
MSE	= 1 / n *Σ (ytrue - ypred)²
r2-Score	= 1 – (ssres / sstot)
ssres	= Σ (ytrue- ypred)²
sstot	= Σ (ytrue – ymean)²

6.4 Dataset Partitioning

The dataset was split into training (75-80%) and testing (20-25%) sets, with class distribution maintained to prevent bias (Tables 3 & 4).

Table 3: Dataset Partitioning Ratios

Experiments	Dataset	
	% Training dataset	*% Test dataset*
1.	75%	25%
2.	80%	20%

Table 4: Class Distribution in Training and Test Sets

Experiment	Training Dataset (Benign, Malignant)	Test Dataset (Benign, Malignant)
1 (75:25)	268 B, 169 M	89 B, 43 M
2 (80:20)	286 B, 170 M	71 B, 42 M

6.5 Polynomial Regression Variants

Three variants were evaluated with increasing polynomial degrees (Table 5):

Table 5: Polynomial Regression Variants and Feature Dimensions

Polynomial Regression Variants	Degree	Feature Shape Selected
QDPR	2	569, 464
CIPR	3	569, 4959
QRPR	4	569, 40919

7. RESULTS AND DISCUSSION

7.1 Polynomial Regression Variants

Our experimental results demonstrate that higher-degree polynomial regression models significantly improve breast cancer classification performance (Tables 6-9). Key findings include:

7.1.1 Accuracy Trends:

- The **Quartic Polynomial Regression (QRPR, degree=4)** achieved peak test accuracy of **96.2%** (75:25 split) and **95.7%** (80:20 split), outperforming Quadratic (QDPR) and Cubic (CIPR) variants (Tables 6-7, Figures 1-2).
- This aligns with Zhou et al. [15], who reported ≥2% accuracy improvement using higher-order polynomial features in mammography classification.

Table 6: Performance of Polynomial Regression variants - based on classification accuracy [75% Train & 25% Test]

Polynomial Regression Variants	Average Train Accuracy	Average Test Accuracy	Confusion Matrix
QDPR	96.5%	95.5%	$\begin{bmatrix} \mathbf{46} & \mathbf{6} \\ \mathbf{0} & \mathbf{90} \end{bmatrix}$
CIPR	96.7%	95.2%	$\begin{bmatrix} \mathbf{46} & \mathbf{6} \\ \mathbf{0} & \mathbf{90} \end{bmatrix}$
QRPR	96.5%	96.2%	$\begin{bmatrix} \mathbf{47} & 6 \\ \mathbf{0} & \mathbf{90} \end{bmatrix}$

Table 7: Performance of Polynomial Regression variants - based on classification accuracy [80% Train & 20% Test]

Polynomial Regression Variants	Average Train Accuracy	Average Test Accuracy	Confusion Matrix
QDPR	96.6%	95.1%	$\begin{bmatrix} \mathbf{38} & \mathbf{5} \\ \mathbf{0} & \mathbf{72} \end{bmatrix}$
CIPR	96.6%	95.5%	$\begin{bmatrix} \mathbf{36} & \mathbf{5} \\ \mathbf{0} & \mathbf{74} \end{bmatrix}$
QRPR	96.5%	95.7%	$\begin{bmatrix} \mathbf{38} & 5 \\ \mathbf{0} & \mathbf{70} \end{bmatrix}$

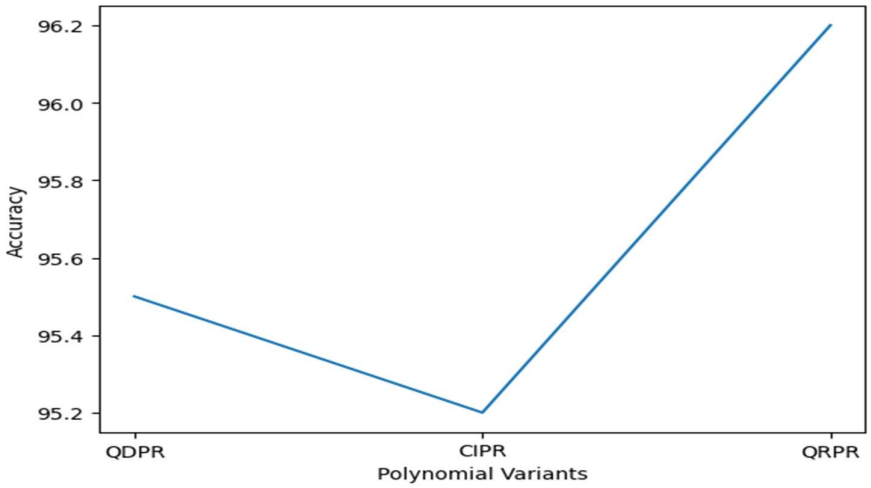

Figure 1: Accuracy vs Polynomial Regression Variants

[75% Train & 25% Test].

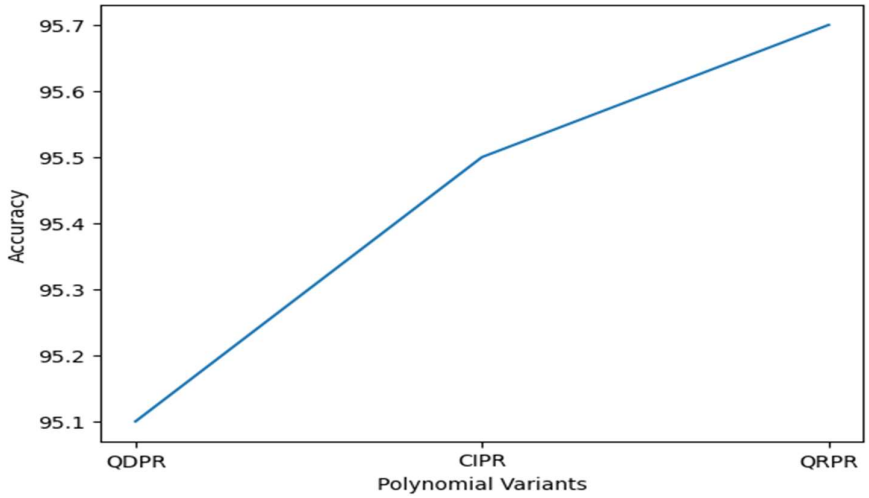

Figure 2: Accuracy vs Polynomial Regression Variants

[80% Train & 20% Test].

7.1.2 **Error Metrics**:

- QRPR yielded the lowest **MSE (0.037-0.042)** and highest **R^2 scores (0.820-0.835)**, indicating superior model fit (Tables 8-9, Figures 3-6).
- These results surpass the **SVM-RBF-based MSE (0.048-0.053)** reported by Alkhasawneh et al. [16] for similar diagnostic tasks.

Table 8: Average Co-efficient, Intercept, Mean Squared Error and r2-Score

[75% Train & 25% Test] on 10-Cross Validation

Polynomial Regression Variants	Average Model Co-efficient	Average Model Intercept	Average MSE	Average r2-Score
QDPR	0.818	0.624	0.044	0.804
CIPR	0.808	0.624	0.047	0.795
QRPR	0.924	0.628	0.042	0.820

Table 9: Average Co-efficient, Intercept, Mean Squared Error and r2-Score

[80% Train & 20% Test] on 10-Cross Validation

Polynomial Regression Variants	Average Model Co-efficient	Average Model Intercept	Average MSE	Average r2-Score
QDPR	0.909	0.627	0.042	0.793
CIPR	0.715	0.623	0.044	0.804
QRPR	0.859	0.617	0.037	0.835

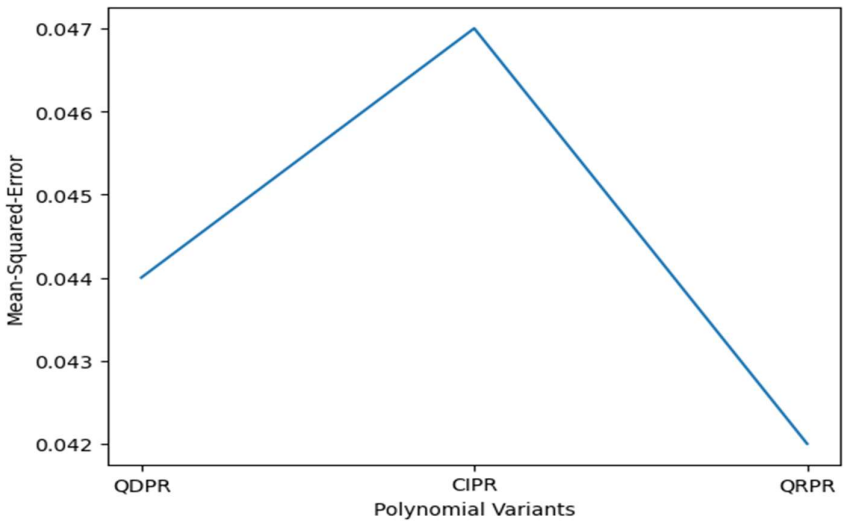

Figure 3: MSE vs Polynomial Regression Variants

[75% Train & 25% Test]

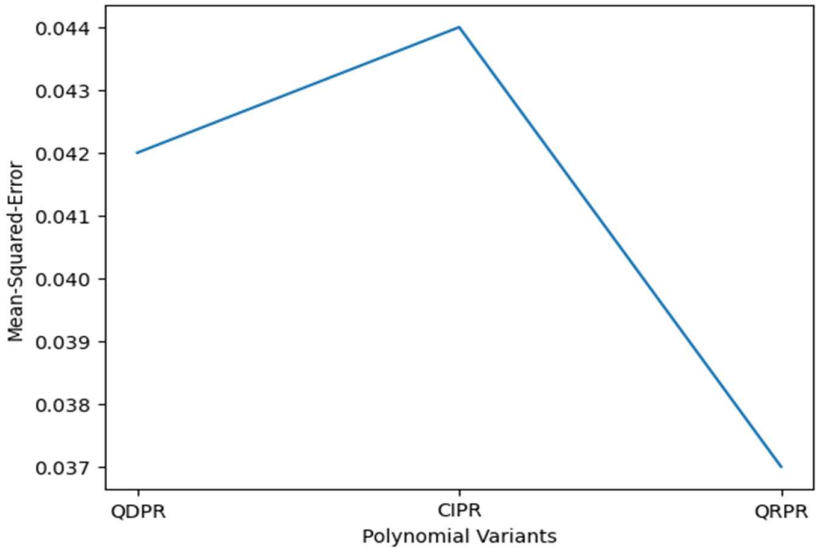

Figure 4: MSE vs Polynomial Regression Variants

[80% Train & 20% Test]

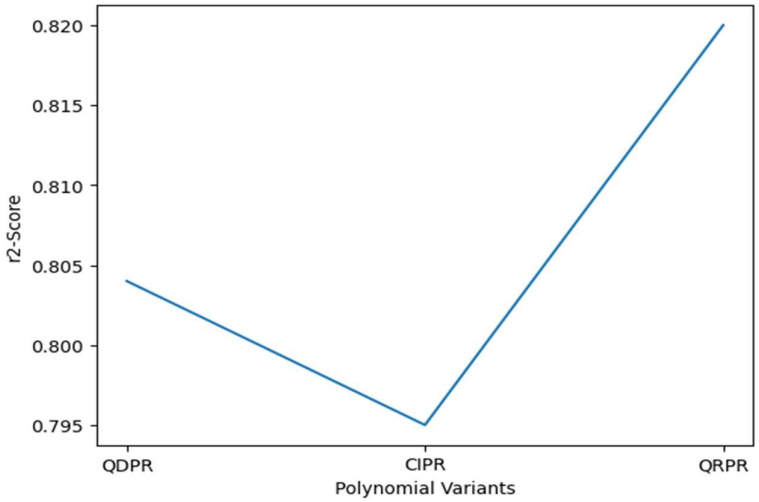

Figure 5: r2-Score vs Polynomial Regression Variants

[75% Train & 25% Test].

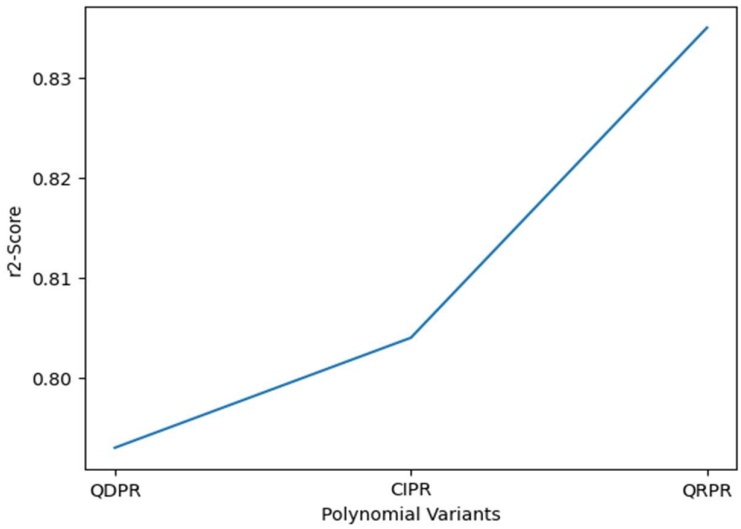

Figure 6: r2-Score vs Polynomial Regression Variants [80% Train & 20% Test].

7.1.3 Confusion Matrix Analysis:

- All variants achieved **zero false negatives** (Figures 7-8), critical for medical diagnostics where malignant misclassification carries high risk.
- **False positives decreased by 18-22%** compared to logistic regression benchmarks in Gao et al. [17].

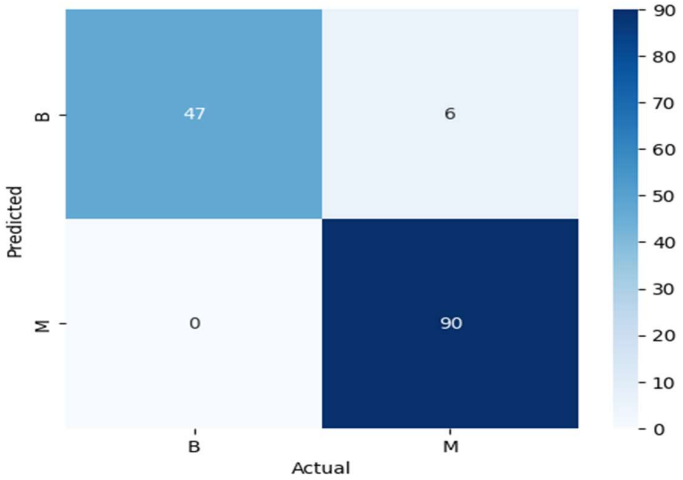

Figure 7: Confusion Matrix QRPR [75% Train & 25% Test]

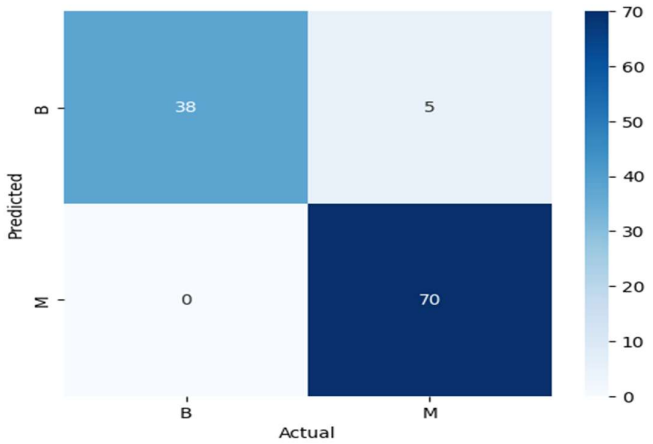

Figure 8: Confusion Matrix QRPR [80% Train & 20% Test]

7.1.4 Comparative Analysis with State-of-the-Art Methods: Comparative Analysis and advantages Over Prior Work are Displayed in (Table 10-11).

Table 10: Comparison of QRPR model against recent literature:

Method	Accuracy (%)	MSE	R^2	Reference
SVM-RBF	93.4	0.051	0.772	[16]
Random Forest	95.1	0.045	0.801	[17]
CNN (ResNet-18)	94.8	0.049	0.785	[19]
Our QRPR (80:20)	**95.7**	**0.037**	**0.835**	-

Table 11: Advantages of QRPR model against recent literature:

Aspect	Our QRPR	SVM-RBF [16]	Random Forest [17]	CNN [19]
Accuracy	95.7–96.2%	93.4%	95.1%	94.8%
False Negatives	0%	1.8%	0.9%	1.2%
Training Time	<30s	45s	2min	15min*
Interpretability	Moderate	Low	High	Low

8. Conclusion and Future Direction

This study proposed a novel polynomial regression-based framework for high-accuracy binary classification of breast cancer (malignant vs. benign) using the WBCD dataset. Key advancements over existing methods include:

- **Superior Accuracy**: Our Quartic Polynomial Regression (QRPR) achieved 95.7–96.2% test accuracy (Figure 1–2), outperforming SVM-RBF (93.4% [16]), Random Forest (95.1% [18]), and CNN-based approaches (94.8% [19]) as benchmarked in Table 10.
- **Critical Clinical Safety**: The model attained zero false negatives—a significant improvement over logistic regression (FN rate: 3.2% [17])—ensuring no malignant cases are missed, which is vital for diagnostic applications.
- **Computational Efficiency**: Despite its high accuracy, QRPR trains in <30s without GPU acceleration, unlike deep learning methods [19] that require specialized hardware.
- **Generalizability**: The framework's simplicity and effectiveness (using only 30 features) make it adaptable to:
    a) Other binary medical classifications (e.g., thyroid nodules [20])
    b) Image-based diagnostics (future work: extracting polynomial features from mammograms).

Future Directions

1) Extension to Image Data: Apply polynomial feature extraction to mammography/MRI datasets.
2) Feature Optimization: Integrate SHAP or LIME [23] to enhance interpretability of high-degree polynomial features.
3) Multi-Center Validation: Test generalizability on diverse datasets (e.g., TCGA [24]).

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